Problem Set 1
Optical Waveguides and Fibers (OWF)

will be discussed in the tutorial on October 28, 2015

Exercise 1: Long-haul transmission fiber link

For a transatlantic fiber link, e.g., from London to New York, a distance of 7000 km has to be bridged. This is accomplished using optical fibers. Each fiber carries multiple wavelength channels within a total bandwidth that is given by the so-called C-band, which ranges from 1530 nm to 1560 nm. A single wavelength channel transmits data at a bitrate of 100 Gbit/s. The spectral separation between two adjacent channels is 50 GHz. The optical fiber used for this link exhibits a loss of 0.2 dB/km within the C-band. These losses are overcome by amplifiers with a gain of $G = 20\,\mathrm{dB}$, where the gain in dB is defined by:

$$G = 10 \log \left\{ \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \right\}.$$

In this equation P_{in} and P_{out} are the input and output power of the amplifier, respectively. One amplifier is used per fiber and all channels are amplified simultaneously. Each amplifier needs electrical power supply. The power efficiency of the amplifier, i.e., the ratio between the total optical output power and the electrical input power amounts to $\eta = 0.01$.

a) How many channels can be used in one fiber? What is the total datarate of the link assuming that it contains 25 fibers?

   Hint: Conversion between absolute frequency and wavelength: $f = \frac{c}{\lambda}$. The bandwidth Δf in terms of frequency can be related to bandwidth $\Delta\lambda$ in term of wavelength using: $\Delta f = \frac{df}{d\lambda} \cdot \Delta\lambda$.

b) Estimate the total attenuation of an optical signal assuming that there is no amplification along the transmission link. Give the result in logarithmic as well as linear units.

   Hint: $a_{[\mathrm{dB}]} = 10 \cdot \log \left(a_{[\mathrm{lin}]}\right)$.

c) How many photons would you have to launch into the optical fiber in London in order to receive a single photon in New York if not using an optical amplifier? How much energy would be needed to generate this many photons? Compare this energy to the available energy of our universe.

   Hint: The (observable) universe is estimated to be a sphere with a radius of 1.4×10^{10} light years, and a mean density of $3 \times 10^{-30}\,{}^{\mathrm{g}}\!/_{\mathrm{cm}^3}$.

d) To keep the power distribution along the link as uniform as possible, which distance would you suggest between two amplifiers? How many amplifiers are needed for the link?

e) Estimate the power consumption assuming that each wavelength channel should carry a power of 0 dBm at the output of each amplifier. Assume that electrical line losses can be neglected.

   Hint: $P_{[\mathrm{dBm}]} = 10 \log \left\{ \frac{P_{[\mathrm{lin}]}}{1\mathrm{mW}} \right\}$.

Exercise 2: Helmholtz equation for the magnetic field $\underline{\mathbf{H}}$

Assuming a time dependence of the kind $e^{j\omega t}$, and assuming that there are no free charges, no free currents and non-magnetic medium, Maxwell's equations take the following form:

$$\begin{aligned} \nabla \cdot \underline{\mathbf{D}} &= 0 && (1) \\ \nabla \times \underline{\mathbf{H}} &= j\omega\epsilon_0 \underline{\epsilon}_r \underline{\mathbf{E}} && (2) \\ \nabla \times \underline{\mathbf{E}} &= -j\omega\mu_o \underline{\mathbf{H}} && (3) \\ \nabla \cdot \underline{\mathbf{H}} &= 0. && (4) \end{aligned}$$

Vectors are denoted in **bold** and complex quantities are <u>underlined</u>.

a) Derive the vector wave equation for the magnetic field in inhomogeneous media:

$$\nabla^2 \underline{\mathbf{H}} + \frac{\nabla \underline{\epsilon}_r}{\underline{\epsilon}_r} \times (\nabla \times \underline{\mathbf{H}}) + \omega^2 \mu_0 \epsilon_0 \underline{\epsilon}_r \underline{\mathbf{H}} = 0 \tag{5}$$

Hints:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \tag{6}$$

$$\nabla \times (\psi \mathbf{A}) = \nabla \psi \times \mathbf{A} + \psi \nabla \times \mathbf{A} \tag{7}$$

b) Show that if $\nabla \underline{\epsilon}_r = 0$, then $\underline{\mathbf{H}} = \underline{\mathbf{H}}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$ is a solution of Eq. (5) with $\mathbf{k}^2 = \omega^2 \epsilon_0 \underline{\epsilon}_r \mu_0$.

c) Show that $\mathbf{k}$ and $\underline{\mathbf{H}}_0$ must be orthogonal.

d) Assume that $\underline{\mathbf{H}}_0 = H_0 \mathbf{e}_y$ and $\mathbf{k} = k\mathbf{e}_z$, where $\mathbf{e}_y$ ($\mathbf{e}_z$) is the unit vector along y (z). Calculate the corresponding electric field. Observe that the vectors $\underline{\mathbf{E}}$, $\underline{\mathbf{H}}$ and $\mathbf{k}$ of the previous point are mutually orthogonal and form a right-handed triple.

e) If we use the ansatz $\underline{\mathbf{H}} = \underline{\mathbf{H}}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$ in Eq. (5), which condition must be satisfied by the normalized spatial variation of the dielectric constant $\nabla \underline{\epsilon}_r / \underline{\epsilon}_r$ in order that the second term in Eq. (5) is negligible compared to the third term?

Bonus system:

During the term, 3 problem sets will be collected in the tutorial and graded without prior announcement. If for each of these sets more than 70% of the problems have been solved correctly, a bonus of 0.3 grades will be granted on the final mark of the exam.

Questions and Comments:

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